Regular and black hole solutions to higher order curvature Einstein–Yang-Mills–Grassmannian systems in 5 dimensions

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Abstract

Solutions to EYM systems in 5 spacetime dimensions possessing no gravity decoupling limits, feature a peculiar critical behaviour which is absent in their $6, 7$ and 8 dimensional counterparts which do possess flat space limits. This critical behaviour in 5 dimensions persists even when a scalar matter field is added, rendering the model nontrivial in the gravity decoupling limit. To this end, both regular and black hole spherically symmetric solutions to the higher curvature EYM–Grassmannian sigma model model in $d = 5$ spacetime dimensions are constructed. A study of the solutions to the Grassmannian model in flat space is also carried out.

1 Introduction

Gravitational theories in higher dimensions are of current interest in the contexts of (10 dimensional) superstring theory and of theories with large and infinite extra dimensions with non-factorisable metrics. It is thus interesting to study the properties of the corresponding Einstein–Yang-Mills (EYM) systems in higher dimensions generalising the usual four dimensional EYM model whose regular solutions were constructed in [1], and black-hole solutions in [2, 3].

Since the low energy effective action of string theory includes higher order terms in the gravitational and Yang-Mills (YM) curvatures, such systems were recently studied in spacetime dimensions $d = 6, 7, 8$ [4] and $d = 5$ [5], in which dimensions it is necessary to include higher curvature YM terms to enable the existence of particle like solutions.

As expected, the gravitating YM models in spacetime dimensions $d = 5, 6, 7, 8$ support particle like solutions for a finite range of the gravitational parameter α^2, say to α^2_{max}. This is similar to the case of gravitating monopoles [6, 7] in $d = 4$. What actually happens here differently from the latter case is that two solutions exist for a given value of α^2. But there is a marked difference between the $d = 6, 7, 8$ cases [4], where there exist two solutions for all values of α^2, and the $d = 5$ case [5] where this is true for values of α^2 up to a critical value α^2_c. In the latter case, solutions exist for values of α^2 oscillating about α^2_c. The tracking of this peculiar singular behaviour in the $d = 5$ model is the aim of the present work.

Now the most obvious difference between the solutions of the models [4] in $d = 6, 7, 8$ on the one hand, and those of the model [5] in $d = 5$ on the other, is that the solutions in the former cases persist in the gravity decoupling limit [1], while those in the $d = 5$ model do not [2]. We have therefore added a scalar matter field to the $d = 5$ model, which renders the gravity decoupling limit nontrivial, to investigate the nature of the critical behaviour discovered in [5]. We thus answer the question: Is this critical behaviour a consequence of the absence of a gravity decoupling limit in $d = 5$, or is it a peculiarity of the dimensionality of the spacetime itself? Our answer is, that this property pertains to the dimensionality of the spacetime, since we find that it persists also in the new model which supports a gravity decoupling solution.

For this purpose we introduce an $SU(2)$ gauged 4×2 Grassmannian field [10] describing a sigma model, to the $d = 5$ EYM model. The introduction of this scalar field is analogous to the inclusion of a Higgs[3] field in $d = 4$ as in Refs. [6, 7].

[1]The solutions in the flat space limit of the $d = 6, 7, 8$ models [4] are exemplified by the instanton–like solution studied in [8].

[2]Note that due the Derrick scaling requirement the YM field in the static 4 Euclidean dimensions supports a 'soliton' only if the YM system consists of the $p = 1$ term, exclusively. On the other hand when gravity is switched on, the absence of the $p = 2$ YM term (see Ref. [9] for the YM hierarchy) prevents the existence of a soliton, due to the same scaling requirement.

[3]The choice of a Grassmannian field rather than a Higgs field is because the gauge connection in a $d - 1$ dimensional Higgs model [12] supporting a 'soliton' behaves as *one half pure gauge* asymptotically, resulting in r^{-1} decay. As a result the integral of the $p = 1$ YM term is convergent only in $(d-1) \leq 3$ so that only higher p YM terms are admissible for $(d-1) \geq 4$. So if we insist in keeping the usual $p = 1$ YM term, then we must avoid using of a (generalised) Higgs model [12]. This contrasts with the faster decay of an 'instanton' in a pure gauge theory where the gauge connection is *pure gauge* and hence the integral of the $p = 1$ YM term is convergent in $d - 1 = 4$. It turns out that the connection of the gauged Grassmannian model [10] is asymptotically *pure*

We find that the singular behaviour in question, which is peculiar to the solutions in $d = 5$ spacetime only, presists whether or not the model supports a regular solution in the flat space limit.

2 The model and the equations

In the first subsection we give the Lagrangian of our 4+1 dimensional model in Minkowski space, and in the second one we impose static spherically symmetry and write down the resulting one dimensional ordinary differential equations.

2.1 The models

We take the gravitational and YM sectors of the model to be precisely the one analysed in [5], augmented by the Grassmannian sigma model term

$$\mathcal{L} = \mathcal{L}_{grav} \ + \ \mathcal{L}_{YM} \ + \ \mathcal{L}_{grass} \ , \tag{1}$$

in 5 spacetime dimensions. The first two terms of (1) describe the EYM sector defined as

$$\mathcal{L}_{grav} = e \ \frac{\kappa_1}{2} \ R_{(1)} \quad , \quad \mathcal{L}_{YM} = e \ \left(\frac{\tau_1}{4} \ \mathrm{Tr} \ F(2)^2 + \frac{\tau_2}{48} \ \mathrm{Tr} \ F(4)^2 \right) \ , \tag{2}$$

where $R_{(1)}$ describes the usual Einstein gravity, and $e = \det e^a_\mu = \sqrt{-\det g_{\mu\nu}}$, e^a_μ are the 5-beins. $F(2p)$, for $p = 1, 2$, is the $2p$-form YM curvature (see Refs. [4] and [9]), which for $p = 1$ is the usual 2-form YM curvature taking values in the antihermtian representation of the algebra of $SU(2)$ here.

The Grassmannian sigma model part of (1) is

$$\mathcal{L}_{grass} = e \, \mathrm{Tr} \left(\tau D_\mu z^\dagger D^\mu z + \frac{1}{2} \tau \mu^2 (\mathbb{1} + z^\dagger \Gamma_5 z) + \lambda (\mathbb{1} - z^\dagger z) \right) \tag{3}$$

described by the 4×2 Grassmannian field subject to the 2×2 condition

$$z^\dagger z = \mathbb{1}$$

and whose $SU(2)$ covariant derivative is defined by

$$D_\mu z = \partial_\mu z - z \, A_\mu \ . \tag{4}$$

In (3) the constant μ has the dimensions of mass and leads to the exponential localisation of the ensuing topologically stable lump, and the Lagrange multiplier λ is a 2×2 array. The gravity decoupled version of this model is the gauged Grassmannian sigma model for which the 'instanton' solutions were constructed numerically in [11], which will be studied in more detail here.

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gauge, and hence our choice.

2.2 The classical equations

In d dimensional spacetime, we restrict to static fields that are spherically symmetric in the $d-1$ spacelike dimensions with the metric Ansatz

$$ds^2 = -\sigma(r)^2 N(r) dt^2 + N(r)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 \tag{5}$$

where r is the spacelike radial coordinate and $d\Omega_{d-2}$ is the $d-2$ dimensional angular volume element.

We take the static spherically symmetric $SU(2)$ YM field in 5 spacetime (i.e. 4 Euclidean) dimensions, in one or other chiral representation of $SO_{\pm}(4)$, to be

$$A_0 = 0 \ , \quad A_i = \left(\frac{1-w}{r}\right) \Sigma_{ij}^{(\pm)} \hat{x}_j \ , \quad \Sigma_{ij}^{(\pm)} = -\frac{1}{4}\left(\frac{1 \pm \gamma_5}{2}\right) [\gamma_i, \gamma_j] \ , \tag{6}$$

where

$$\Sigma_{ij}^{(\pm)} = -\frac{1}{4} \Sigma_{[i}^{(\pm)} \Sigma_{j]}^{(\pm)}$$

where in a more familiar notation $\Sigma_i^{(+)} = \sigma_i = (i\vec{\sigma}, 1\!\!1)$ and $\Sigma_i^{(-)} = \tilde{\sigma}_i = (-i\vec{\sigma}, 1\!\!1)$, with $i = 1, 2, 3, 4$ and in terms of the three Pauli spin matrices $\vec{\sigma}$.

The spherically symmetric Ansatz for the Grassmannian field z, whose consistency has been checked, is

$$z = \begin{bmatrix} \sin\frac{f}{2} & 1\!\!1 \\ \cos\frac{f}{2} & \hat{x}_i \tilde{\sigma}_i \end{bmatrix} \ . \tag{7}$$

Subjecting (2) and (3) to spherical symmetry by empoloying the Ansätze (5), (6) and (7), and subjecting the resulting one dimensional Lagrange density to the variational principle, we find the following equations for the functions $f(r)$, $w(r)$, $N(r)$ and $\sigma(r)$,

$$(r^3 \sigma N f')' + r\,\sigma \left(3w - \mu^2 r^2\right) \sin f = 0, \tag{8}$$

$$\tau_1 \left((r\sigma N w')' - 2r^{-1}\sigma(w^2-1)w\right) + 3\tau_2 (w^2-1) \left(r^{-3}\sigma N (w^2-1) w'\right)' = 2\tau\, r\, \sigma (w + \cos f), \tag{9}$$

$$\begin{aligned} m' &= \frac{1}{8} r \left(\tau_1 \left[N w'^2 + \left(\frac{w^2-1}{r}\right)^2 \right] + \frac{3}{r^2} \tau_2 \left(\frac{w^2-1}{r}\right)^2 N w'^2 \right) \\ &+ \frac{\tau}{12} r^3 \left[N f'^2 + \frac{3}{r^2} \left(w^2 + 2w\cos f + 1\right) + 2\mu^2 (1 - \cos f) \right], \end{aligned} \tag{10}$$

$$\kappa_1 \left(\frac{\sigma'}{\sigma}\right) = \frac{n_5}{8r} \left[\tau_1 + \frac{3}{r^2} \tau_2 \left(\frac{w^2-1}{r}\right)^2 \right] w'^2 + \frac{n_5}{12} \tau\, r f'^2 \ . \tag{11}$$

The ADM mass $M = \lim_{r\to\infty} m(r)$, with $m(r)$ defined as

$$m(r) = n_5^{-1}\kappa_1 r^2(1-N) \ . \qquad (12)$$

For $N(r) = \sigma(r) = 1$ everywhere, namely the gravity decoupling limit, (8) and (9) satisfy the flat space solution.

In the numerical work below, we set the self interaction potential of the Grassmannian field, $\mu^2 = 0$ without changing the qualitative nature of the solutions. This is because in this model, as in Higgs models, the finite energy condition leads to a unique asymptotic value for the matter field, as can be seen easily by inspection of the term multiplying τ in (10). This situation contrasts with that in certain other gauged nonlinear sigma models, where a unique asymptotic value for the matter field can be ensured only by the inclusion of a self interaction potential, e.g. the pion mass potential of the Skyrme model, leading to bifurcations [13].

2.3 Boundary values and asymptotic behaviour

In the next section, we will solve the above equations with the appropriate boundary conditions for the radial functions $m(r)$, $\sigma(r)$ $w(r)$ and $f(r)$ which guarantee the solution to be regular at the origin and to have finite energy. For regular solutions, the boundary conditions at the origin are

$$m(0) = 0 \ \ , \qquad w(0) = 1, \qquad f(0) = \pi, \qquad (13)$$

while the conditions satisfied on the event horizon $r = r_h$ are

$$N(r_h) = 0, \quad \sigma(r_h) = \sigma_h, \quad w(r_h) = w_h, \quad f(r_h) = f_h, \qquad (14)$$

with σ_h, w_h, f_h real constants. The asymptotic form of the solution is

$$\lim_{r\to\infty} \sigma(r) = 1 \ \ , \qquad \lim_{r\to\infty} w(r) = -1 \qquad , \lim_{r\to\infty} f(r) = 0. \qquad (15)$$

The condition on $\sigma(r)$ results in the metric being asymptotically Minkowskian.

The asymptotic solutions to these functions can be systematically constructed in both regions, near the origin (or event horizon) and for $r \gg 1$. Defining $\alpha^2 = \frac{n_d}{8\kappa_1}$ we find for $r \ll 1$

$$\begin{aligned} f(r) &= \pi - c_3 r + o(r^3) \ \ , \\ w(r) &= 1 + c_1 r^2 + o(r^4) \ \ , \\ \sigma(r) &= \sigma_0[1 + 2\alpha^2 c_1^2 r^2(\tau_1 + 12\tau_2 c_1^2] + o(r^4)) \ \ , \\ m(r) &= \frac{1}{4} r^4[c_1^2(\tau_1 + 6c_1^2\tau_2) + \frac{1}{6}\tau c_3^2] + o(r^6). \end{aligned} \qquad (16)$$

For black hole configurations, the expression of the solutions near the event horizon is

$$\begin{aligned} f(r) &= f_h + f'(r_h)(r-r_h) + O((r-r_h)^2), \\ w(r) &= w_h + w'(r_h)(r-r_h) + O((r-r_h)^2), \\ \sigma(r) &= \sigma_h\alpha^2\left(1 + \left((\tau_1 + \frac{3\tau_2(w_h^2-1)^2}{r_h^4})w'^2(r_h) + \frac{2}{3}\tau r_h f'^2(r_h)\right)(r-r_h)\right) + O((r-r_h)^2), \\ m(r) &= \frac{r_h^2}{8\alpha^2} + m'(r_h)(r-r_h) + O((r-r_h)^2), \end{aligned} \qquad (17)$$

where

$$
\begin{aligned}
m'(r_h) &= \frac{\tau_1(w_h^2-1)^2}{8r_h} + \frac{\tau r_h^3}{12}\left(\frac{3}{r_h^2}(w_h^2+2w_h\cos f_h+1)+2\mu^2(1-\cos f_h)\right), \\
w'(r_h) &= \frac{2\tau r_h(w_h+\cos f_h)+2\tau_1 w_h(w_h-1)/r_h}{(\tau_1 r_h+3\tau_2(w_h^2-1)^2/r_h^3)(-8\alpha^2 m'(r_h)/r_h^2+2/r_h)}, \\
f'(r_h) &= -\frac{r_h(3w_h^2-\mu^2 r_h^2)\sin f_h}{r_h^3(-8\alpha^2 m'(r_h)/r_h^2+2/r_h)}\,.
\end{aligned}
\tag{18}
$$

For $r \gg 1$ we find

$$
\begin{aligned}
f(r) &= r^{-1}K_2(\mu r) \quad , \\
w(r) &= -1 + \frac{c_2}{r^{d-3}} \quad , \\
\sigma(r) &= 1 - \frac{\tau_1\alpha^2 c_2^2(d-3)^2}{(2d-4)r^{2d-4}} \quad , \\
m(r) &= m_\infty - \frac{\tau_1(d-3)c_2^2}{8r^{d-1}}
\end{aligned}
\tag{19}
$$

In the first member of (19), $K_2(\mu r)$ is a Bessel function leading to exponential decay by virtue of the mass term μ in the potential. The constants $c_1, c_2, c_3, \sigma_0, f_h, w_h, \sigma_h$ and m_∞ have to be determined numerically; m_∞ is nothing else but the ADM mass of the solution as noted previously. They depend generically on the coupling constants of the theory.

3 Numerical results

In this section we will present three sets of results. These are the non gravitating $SU(2)$ gauged Grassmannian solitons, the gravitating regular solutions of this system, and, the corresponding black hole solutions. Each will be reported in a separate subsection.

The numerical integrations were carried out using both a shooting method as well as applying the numerical program COLSYS [14], with complete agreement to very high accuracy.

3.1 $SU(2)$ gauged Grassmannian solitons

Since an important reason for considering the model in this work is that it supports a non gravitating flat space limit, it is worth studying the properties of the latter for its own sake. This is especially so since we find that this (non gravitating) model supports solutions which exhibit nodes in the profile of the Grassmann function $f(r)$ defined in (7).

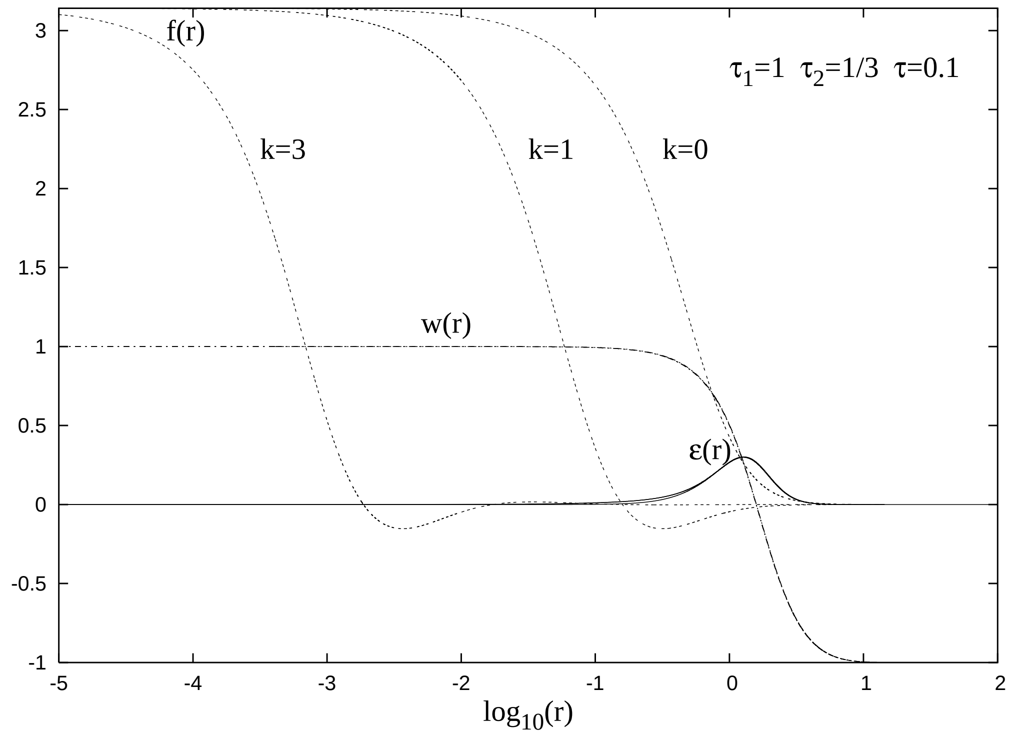

Figure 1. The functions $w(r)$ and $f(r)$ and the energy density $\epsilon(r)$ are plotted as functions of radius for typical flat space solutions with the coupling constants $\tau_1 = 3\tau_2 = 1, \tau = 0.1$. The node number k of the Grassmanian function $f(r)$ is also indicated.

The profiles of the YM functions $w(r)$ do not change appreciably for the solutions with different number of nodes of $f(r)$, neither qualitatively nor quantitatively, exhibiting only one node. Also, somewhat unexpected, the total mass/energy of these solutions stays almost constant, when increasing the node number of $f(r)$ (with differences less than one percent). We shall see below that all the multinode solutions result in qualitatively very similar gravitating solutions, which we will exploit to simplify the numerical work section (3.3).

It turns out that there are solutions with infinitely many nodes in the function $f(r)$, such that the profiles contract towards the origin as the number of nodes n increases. The profiles of $w(r)$ and the energy density remain insensitive to the increase in the number of nodes n of $f(r)$, as seen in Figure 1. In the limit $n \to \infty$ the profile of $f(r) \to f_\infty(r)$ shrinks to the origin such that

$$f_\infty(r) = 0 \qquad \text{for all} \quad r\,.$$

We refer to this configuration as that of frozen f, namely $f = 0$ everywhere. Also, for all considered solutions, as well as the gravitating counterparts, we find that the gauge function $w(r)$ monotonically decreases towards its asymptotic value without presenting local extrema.

3.2 Regular gravitating solutions

We numerically integrate the Eqs. (8)-(11) with the boundary conditions (13), (15) for $\tau_1 = 1, \tau_2 = 1/3$ and several values of τ, finding the following picture. First, for α^2 being small enough, a branch of solutions smoothly emerges from the flat space configurations. When α^2 increases, the mass parameter M decreases, as well as the value $\sigma(0)$ and the minimum N_m

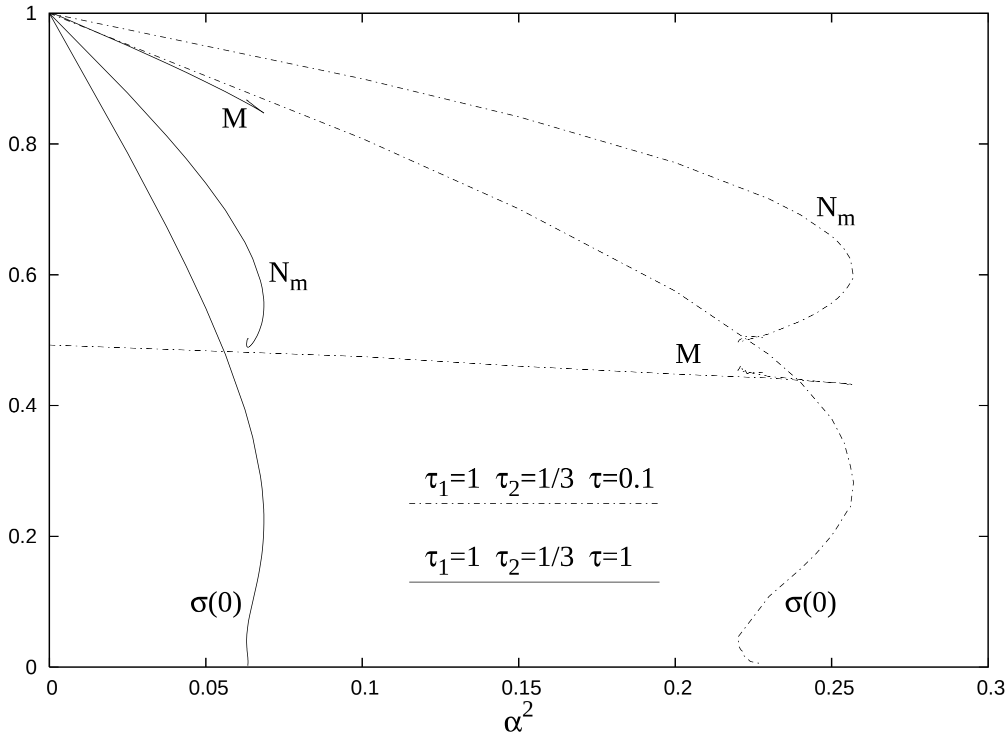

Figure 2. The value N_m of the minimum of the metric function N, the value of the metric function σ at the origin $\sigma(0)$, as well as the mass M are shown as a function of $\alpha^2 = n_d/(8\kappa_1)$ for regular solutions with $\tau_1 = 3\tau_2 = 1$ and two different values of τ. The mass of the flat space solution is $M = 1.00236$ for $\tau = 1$ and $M = 0.49257$ ($\tau = 0.1$).

of the function $N(r)$ decrease, as indicated in Fig. 2. These solutions exist up to a maximal value α_{max} of the parameter α, which is smaller than the corresponding value in the pure EYM theory [5], and depends on the value of the coupling parameter τ. For example, we find numerically $\alpha^2_{max} \approx 0.2573$ for $\tau = 0.1$ while the corresponding value for $\tau = 1$ is $\alpha^2_{max} \approx 0.06855$. (Without a Grassmanian field, this branch extends up to $\alpha^2_{max} \approx 0.5648$.)

Similar to the EYM case [5], we found another branch of solutions on the interval $\alpha^2 \in [\alpha^2_{cr(1)}, \alpha^2_{max}]$ with $\alpha^2_{cr(1)}$ depending again on the value of τ (e.g. $\alpha^2_{cr(1)} \approx 0.06302$ for $\tau = 1$). On this second branch of solutions, both $\sigma(0)$ and N_m continue to decrease but stay finite. However, a third branch of solutions exists for $\alpha^2 \in [\alpha^2_{cr(1)}, \alpha^2_{cr(2)}]$, on which the two quantities decrease further. A fourth branch of solutions has also been found, with a corresponding $\alpha^2_{cr(3)}$ close to $\alpha^2_{cr(2)}$. Further branches of solutions, exhibiting more oscillations very likely exist but their study is a difficult numerical problem. Progressing on this succession of branches, the main observation is that the value $\sigma(0)$ decreases much faster than that of N_m as illustrated in Fig. 2. The pattern strongly suggests that after a finite (or more likely infinite) number of oscillaions of $\sigma(0)$, the solution terminates into a singular solution with $\sigma(0) = 0$ and a finite value of $N(0)$. As seen in Figure 2, the mass parameters do not increase significantly along these secondary branches.

This is the behaviour observed in [5] for the pure EYM theory. The inclusion of a Grassmanian extra field does not seem to qualitatively change the properties of the system.

In Fig. 3, we present the profiles of the metric functions N and σ for the same value of α^2 on the first and second branch.

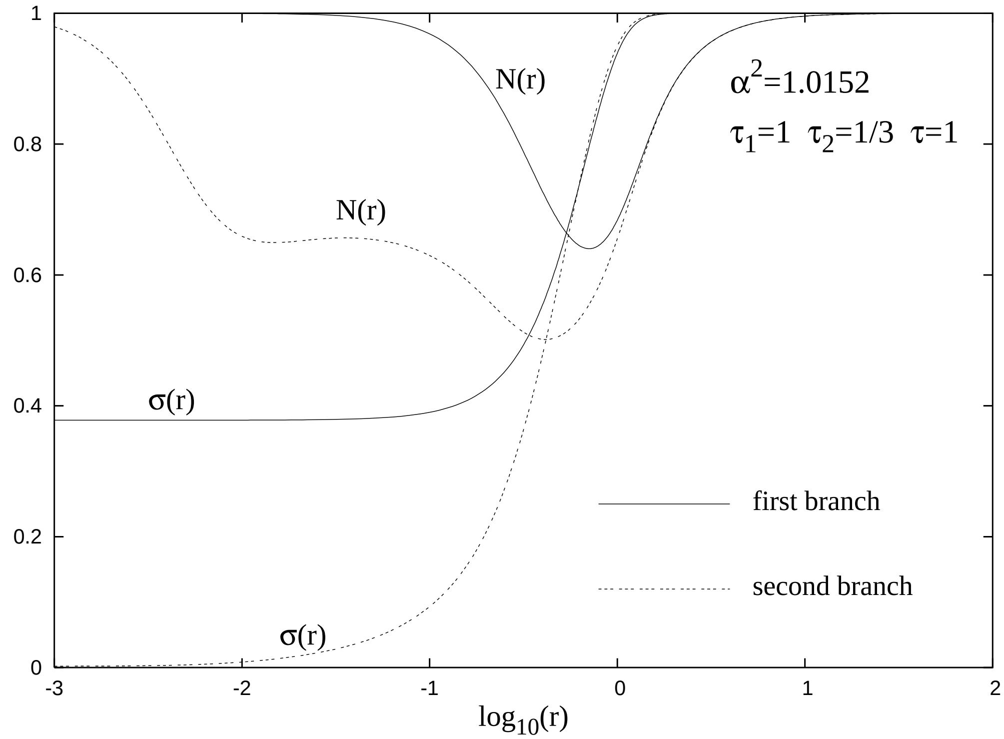

Figure 3. The metric functions $N(r), \sigma(r)$ are plotted as functions of radius for typical regular gravitating solutions.

The discussion in this section is restricted to the Grassmanian field presenting no nodes. We do not expect the consideration of solutions with nodes to change this picture.

3.3 Black hole solutions

According to the standard arguments, one can expect black hole generalisations of the regular configurations to exist at least for small values of the horizon radius r_h. This is confirmed by the numerical analysis for solutions with no nodes in $f(r)$ as well as solutions with a frozen Grassmanian field $f(r) = f_\infty = 0$.

Again, the properties of the solutions we find are rather similar to the five dimensional black hole solutions without a Grassmanian field discussed in [5]. Firstly, black hole solutions seem to exist for all values of α for which regular solutions were constructed. Also, for a given set of couplings (τ_1, τ_2, τ), the solutions exist only for a limited region of the (r_h, α) space.

The typical behaviour of solutions as function of r_h is presented in Figure 4, for a small value of α as compared to to maximal value α_{max} of the regular solutions. Starting from a regular solution and increasing the event horizon radius, we find a first branch of solutions which extends to a maximal value $r_{h(max)}$. The variation of mass and $\sigma(r_h)$ is relatively small on this branch. Extending backwards in r_h, we find a second branch of solutions for $r_h < r_{h(max)}$. This second branch stops at some critical value $r_{h(cr)}$, where the numerical iteration fails to converge. The value of $\sigma(0)$ on this branch decreases drastically, as shown in Fig. 4. Also, the surface gravity κ of the solutions, given by

$$\kappa^2 = -\frac{1}{4} g^{tt} g^{rr} (\partial_r g_{tt})^2,$$

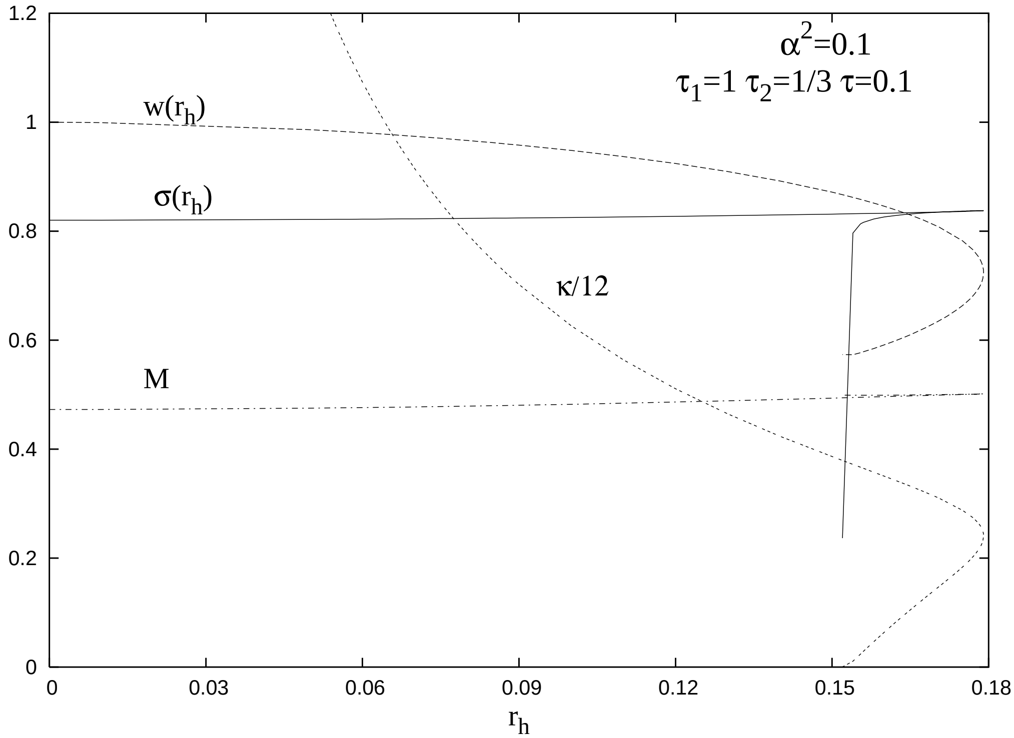

Figure 4. The value w_h of the gauge function at the horizon, the value σ_h of the metric function σ at the horizon, the mass M, as well as the surface gravity κ divided by 12 are shown as functions of the event horizon radius r_h for black hole solutions with $\alpha^2 = 0.1$, $\tau_1 = 3\tau_2 = 1, \tau = 0.1$. These results are obtained for a "frozen" Grassmanian field $f = 0$.

strongly decreases on this branch, approaching a very small value. However, the increase of the total mass is still very small. Similar to the EYM case [5], higher branches of solutions on which the value $\sigma(0)$ continues to decrease further to zero are likely to exist. However, the extension of these branches in r_h will be very small, which makes their study difficult.

Although the results in Figure 4 correspond to a frozen Grassmanian field, we do not expect a different result for solutions with a nontrivial $f(r)$. Similar to the regular case, we find that the node number of the Grassmanian field does not significantly affect the propertiess of the gravitating solutions.

However, the global picture we find (and the corresponding EYM results) may change by considering values of α near α_{max}. We hope to come back on this point in a future work.

4 Summary and discussion

The aim of this work is to find out whether a particular singular behaviour of solutions of a gravitating YM model[4] in $d = 5$ spacetime is a persistent feature of the dimensionality of the spacetime? The reason for asking this question is that the EYM model in question here does not feature this critical behaviour in spacetimes $d = 6, 7, 8$.

More simply stated, the model(s) in $d = 6, 7, 8$ exhibit only a maximum value of the grav-

[4]The definition of the model in question is d (spacetime) dependent. While in all d these are formally the same, nevertheless the gauge groups are different, depending on d [4, 5].

itational coupling $\alpha^2 = \alpha^2_{max}$, while that in $d = 5$ has, in addition to an $\alpha^2 = \alpha^2_{max}$, also an $\alpha^2_{cr} > 0$. This is seen from the oscillatory behaviour of α^2 converging to α^2_{cr} in Figure 2. Because $d = 5$ spacetime is particularly relevant in the context of the AdS/CFT correspondence, and because the solutions in $d = 5$ spacetime differ markedly in this respect from those in $d = 6, 7, 8$ spacetimes, it is important to study this peculiar feature in $d = 5$ further.

The original model(s) introduced in [4] involve a dimensionful constant in addition to the gravitational constant, analogously with the $d = 4$ gravitating YMH model [6, 7]. Like the latter, regular solutions to the higher dimensional EYM models exist only for values of the gravitational coupling α^2 up to a maximum α^2_{max}. But this analogy with the EYMH case goes further only in the $d = 5$ EYM case, where in addition to α^2_{max}, there occurs also a critical $\alpha^2_{cr} > 0$. In the $d = 6, 7, 8$ EYM cases it appears that $\alpha^2_{cr} = 0$. This is surprising since the $d = 6, 7, 8$ EYM models support regular solutions in the flat space limit just like the $d = 4$ EYMH model, while the $d = 5$ EYM model does not support a flat space solution.

It is to throw some light on this question that we modified the $d = 5$ EYM model by introducing a (Grassmannian) scalar matter field, which results in the new model supporting a flat space solution. The result is that the qualitative features of the solutions of the original $d = 5$ EYM model are preserved.

In passing, we studied the (static) solitons of the $d = 5$ flat space $SU(2)$ gauged Grassmannian model, and found that these form an infinite sequence of solutions exhibiting multi-nodes in the profile of the Grassmannian profile function.

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References

[1] R. Bartnik and J. McKinnon, Phys. Rev. Lett. **61** (1988) 141.

[2] M.S. Volkov and D.V. Gal'tsov, JETP Lett. **50** (1989) 346.

[3] P. Bizon, Phys. Rev. Lett. **64** (1990) 2844.

[4] Y. Brihaye, A. Chakrabarti and D.H. Tchrakian, Class. Quantum Grav. **20** (2003) 2765 [hep-th/0202141].

[5] Yves Brihaye, A. Chakrabarti, Betti Hartmann and D.H. Tchrakian, Phys. Lett. **B 561** (2003) 161 [hep-th/0212288].

[6] P. Breitenlohner, P. Forgacs and D. Maison, Nucl. Phys. **B 383** (1992) 357; *ibid.* **442** (1995) 126.

[7] K. Lee, V. P. Nair and E. J. Weinberg, Phys. Rev. **D 45** (1992) 2751.

[8] J. Burzlaff and D. H. Tchrakian, J. Phys. **A 26** (1993) L1053.

[9] see D.H. Tchrakian Yang-Mills hierarchy, Int. J. Mod. Phys. A (Proc.Suppl.) **3A** (1993) 584, and references therein.

[10] R. Manvelyan and D.H. Tchrakian, Phys. Lett. **B 352** (1995) 321; A. Chakrabarti and D.H. Tchrakian, Phys. Lett. **B 376** (1996) 59.

[11] Y. Brihaye, F. Brau and D.H. Tchrakian, Int. J. Mod. Phys. **15** (2000) 251.

[12] G.M. O'Brien and D.H. Tchrakian, Mod. Phys. Lett. **A 4** (1989) 1389; K. Arthur, G.M. O'Brien and D.H. Tchrakian, J. Math. Phys. **38** (1997) 4403.

[13] Y. Brihaye and D.H. Tchrakian, Nonlinearity, 11 (1998) 891; Y. Brihaye, J. Burzlaff, V. Paturyan and D.H. Tchrakian, Nonlinearity, **15** (2002) 385.

[14] U. Asher, J. Christiansen and R. D. Russel, Math. Comput. 33 (1979) 659; ACM Trans. Math. Softw. 7 (1981) 209.